STAT 453: Introduction to Deep Learning and Generative Models

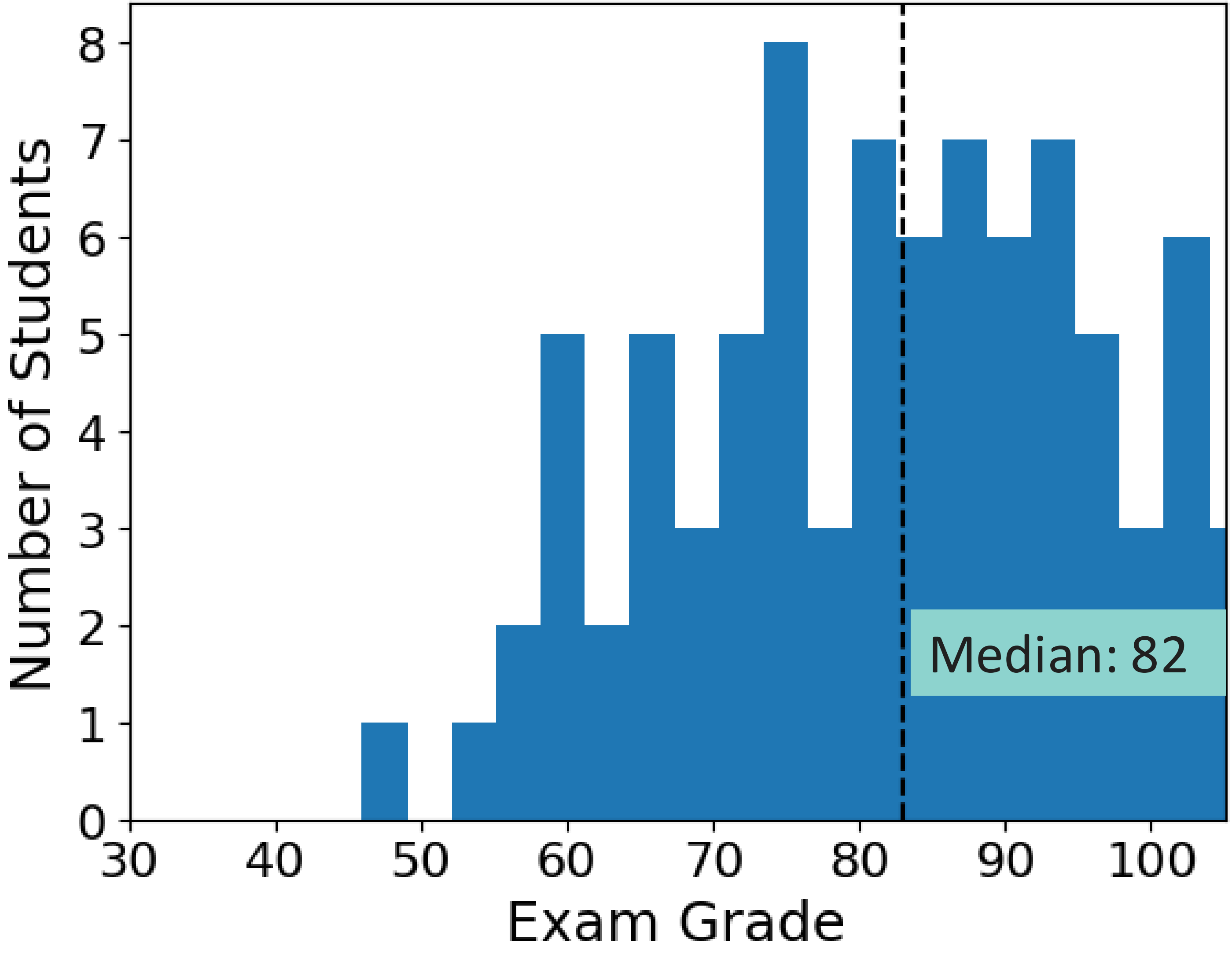
Ben Lengerich

Lecture 15: A Linear Intro to Generative Models

October 27, 2025

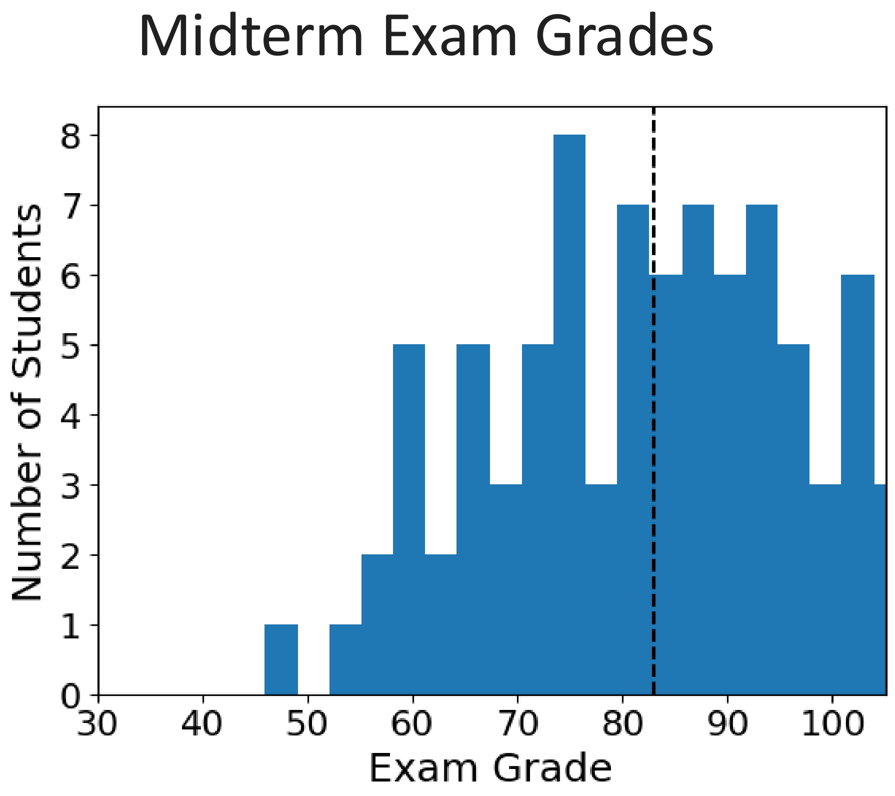
Reading: See course homepage

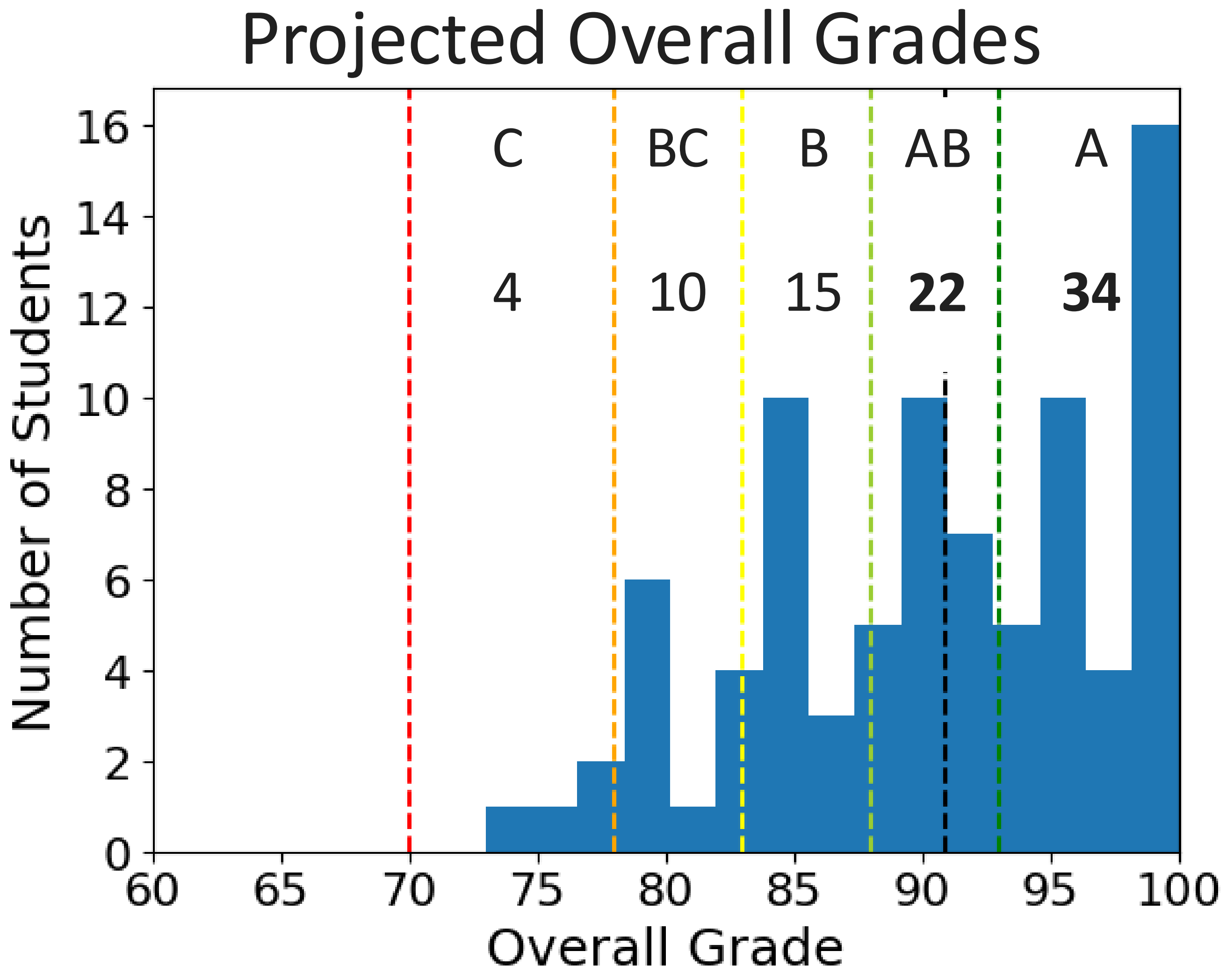
Midterm Exam



Max: 107 (x2)
Median: 82
Mean: 81.5

Midterm Exam in Context





Canvas: put your midterm exam in context

Use Canvas's grades tool to calculate your potential overall grade.

Project Midway Report Project	Nov 7 by 11:59pm	/ 100	
HW4 Assignments	Nov 21 by 11:59pm	/ 100	
HW5 Assignments	Dec 5 by 11:59pm	/ 100	
Project Presentation Project	Dec 10 by 11:59pm	/ 100	
Project Final Report Project	Dec 12 by 11:59pm	/ 300	
Final Exam Exams	Dec 17 by 11:59pm	/ 150	
Lecture Notes (Bonus) Exams	Dec 20 by 11:59pm	25 / 0	
Midterm Exam Adjustment Exams		8 / 0	
Assignments		0%	0.00 / 200.00
Exams		33%	33.00 / 100.00
Project		N/A	0.00 / 0.00
Total		**23.57%**	

Max is 25 → Lecture Notes (Bonus)

Everyone gets 8 → Midterm Exam Adjustment

Course Schedule / Calendar

Week	Lecture Dates	Topic	Assignments
Module 1: Introduction and Foundations			
1	9/3	Course Introduction	
2	9/8, 9/10	A Brief History of DL, Statistics / linear algebra / calculus review	HW1
3	9/15, 9/17	Single-layer networks Parameter Optimization and Gradient Descent	
4	9/22, 9/24	Automatic differentiation with PyTorch, Cluster and cloud computing resources	HW 2
Module 2: Neural Networks			
5	9/29, 10/1	Multinomial logistic regression, Multi-layer perceptrons and backpropagation	
6	10/6, 10/8	Regularization Normalization / Initialization	HW 3
7	10/13, 10/15	Optimization, Learning Rates CNNs	Project Proposal
8	10/20, 10/22	Review, **Midterm Exam**	In-class Exam

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16	12/17	**Final Exam**	Final Exam

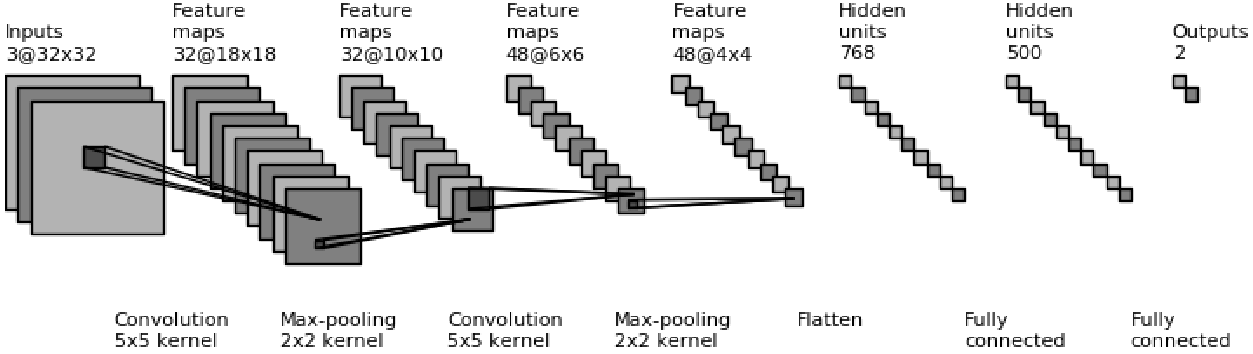
Your Feedback

- Please fill out our anonymous Google Form

Generative Models

Where we're going: Deep Generative Models

Discriminative Model (what we've seen so far)



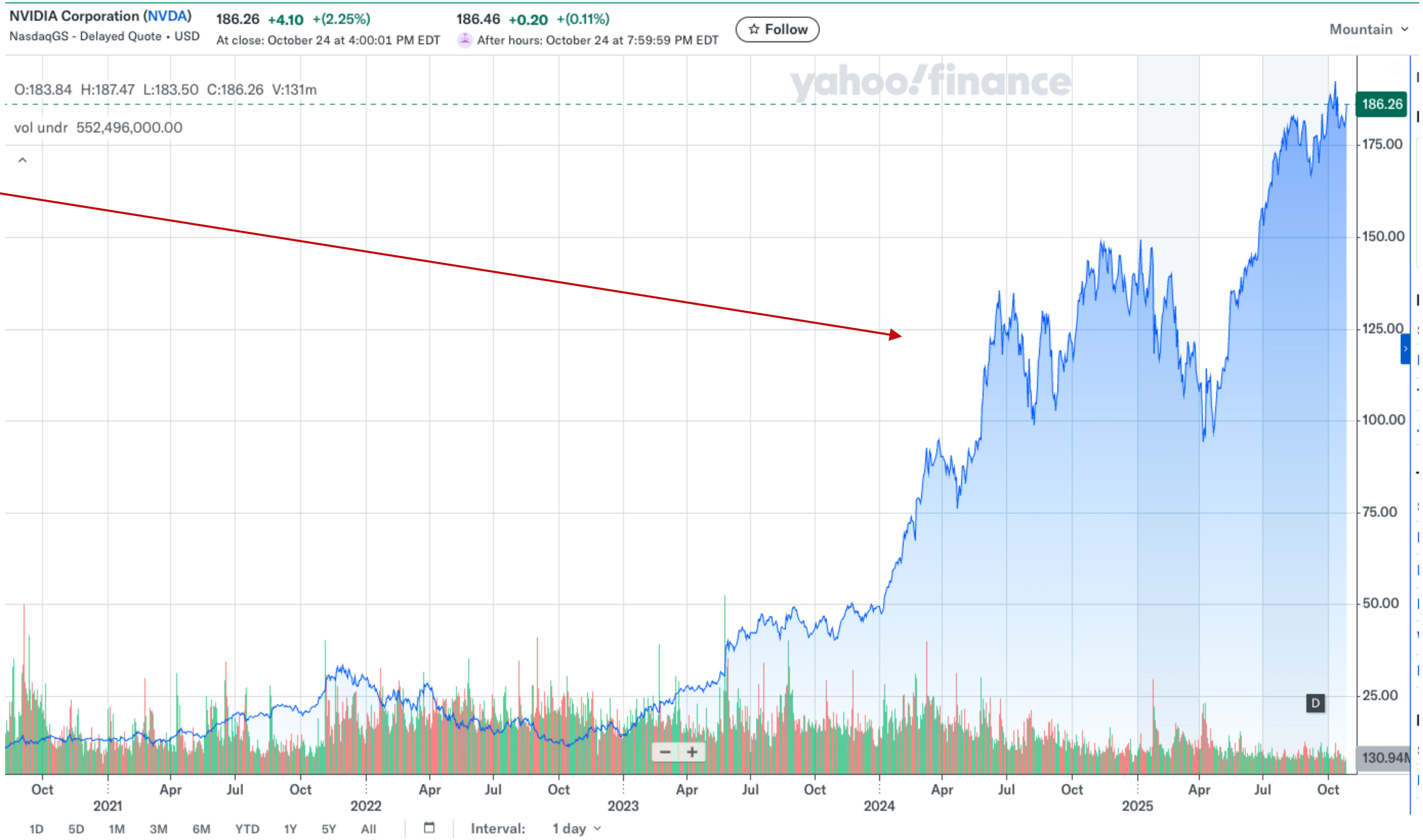
Cat or dog?

Generative Model (what we're going to see)

Gemini Grok deepseek

Where we're going: Deep Generative Models

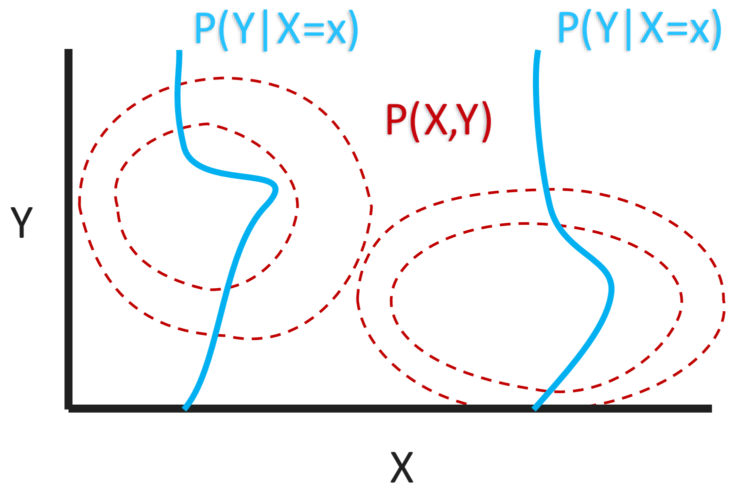
NVIDIA is now valued at **>$4.5T**



A Linear Intro to Generative Models

Generative and Discriminative Models

- **Generative**:
  - Models the joint distribution $\mathrm{P}(X, Y)$.
- **Discriminative**:
  - Models the conditional distribution $\mathrm{P}(Y|X)$.



Two paths to P(Y|X)

- **Discriminative**:

Observe X, Y $\longrightarrow$ Learn $\mathrm{P(Y|X)}$

- **Generative**:
    - Learn $\mathrm{P}(X|Y), \mathrm{P(Y)}$
    - Calculate $P(X) = \int_Y P(X, Y)dY$

Observe X, Y

$$\mathrm{P(Y|X)} = \frac{\mathrm{P(X|Y)P(Y)}}{P(X)}$$

Two paths to classification

- **Discriminative**:

Observe X, Y → $\hat{Y} = \text{argmax}_{\text{Y}} \text{P}(\text{Y}|\text{X})$

- **Generative**:
  - Learn $\text{P}(X|Y), \text{P}(\text{Y})$
  - ~~Calculate~~ $\cancel{P(X) = \int_Y P(X,Y)dY}$

Observe X, Y → $\hat{Y} = \text{argmax}_{\text{Y}} \text{P}(\text{X}|\text{Y})\text{P}(\text{Y})$

Example Discriminative Model: Logistic Regression

- **Discriminative**:

Observe X, Y → Learn P(Y|X)

- Parameterize:
  - $P(Y=1|X) = \sigma(\theta^T X)$, where $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function.
  - $P(Y=0|X) = 1 - P(Y=1|X)$
  - Recall: Why this parameterization?

$$\mathbf{log}\frac{\boldsymbol{P(Y=1|X)}}{\boldsymbol{P(Y=0|X)}} = \log\frac{\sigma(\theta^T X)}{1-\sigma(\theta^T X)}$$

$$= \log\frac{\frac{1}{1+e^{-\theta^T X}}}{1-\frac{1}{1+e^{-\theta^T X}}} = \log\frac{\frac{1}{1+e^{-\theta^T X}}}{\frac{(1+e^{-\theta^T X})-1}{1+e^{-\theta^T X}}} = \log\frac{\frac{1}{1+e^{-\theta^T X}}}{\frac{e^{-\theta^T X}}{1+e^{-\theta^T X}}}$$

$$= \log\frac{1}{e^{-\theta^T X}} = \log e^{\theta^T X} = \boldsymbol{\theta^T X}$$

Example Discriminative Model: Logistic Regression

- **Discriminative**:

Observe X, Y $\longrightarrow$ Learn P(Y|X)

- Parameterize:
  - $P(Y = 1|X) = \sigma(\theta^T X)$, where $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function.
  - $P(Y = 0|X) = 1 - P(Y = 1|X)$
- Estimate $\hat{\theta}$ from observations:
  - $\hat{\theta} = \text{argmax}_\theta \prod_i P(Y_i|X_i;\theta)$
    $= \text{argmax}_\theta \sum_i [Y_i \log \sigma(\theta^T X_i) + (1 - Y_i) \log(1 - \sigma(\theta^T X_i))]$
- Calculate $P(Y = 1|X) = \sigma(\theta^T X)$

Example Generative Model: Naïve Bayes

Learn $\mathrm{P}(X|Y), \mathrm{P}(\mathrm{Y})$

Observe X, Y

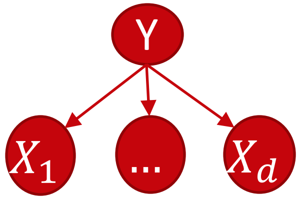
$$\mathrm{P(Y|X)} = \frac{\mathrm{P(X|Y)P(Y)}}{P(X)}$$

- Parameterize:
  - Assume $P(X|Y) = \prod_{j=1}^{d} P(X_j|Y)$,
  - $P(X_j|Y) = N(\mu_{jk}, \sigma_{jk}^2)$

$$P(Y = k) = \frac{\#\, of\ samples\ with\ Y=k}{Total\ samples}$$

Conditional independences of features X | Y

Frequency of labels



Example Generative Model: Naïve Bayes

Learn $\mathrm{P}(X|Y), \mathrm{P}(\mathrm{Y})$

Observe X, Y

$$\mathrm{P}(\mathrm{Y}|\mathrm{X}) = \frac{\mathrm{P}(\mathrm{X}|\mathrm{Y})\mathrm{P}(\mathrm{Y})}{P(X)}$$

- Parameterize:
  - Assume $P(X|Y) = \prod_{j=1}^{d} P\left(X_j \middle| Y\right)$,

$$P(Y = k) = \frac{\#\ of\ samples\ with\ Y{=}k}{Total\ samples}$$

- Estimate:
  - $\hat{\mu}, \hat{\sigma} = \mathrm{argmax}_{\mu,\sigma} P(X|Y)$
- Calculate $P(Y = 1|X) = \dfrac{\prod_{j=1}^{d} P\left(X_j \middle| Y = 1\right) P(Y=1)}{P(X)}$

Summary

- **Discriminative**:

Observe X, Y → Learn $\mathrm{P(Y|X)}$

- **Generative**:

  - Learn $\mathrm{P}(X|Y), \mathrm{P(Y)}$
  - Calculate $P(X) = \int_Y P(X,Y)dY$

Observe X, Y

$$\mathrm{P(Y|X)} = \frac{\mathrm{P(X|Y)P(Y)}}{P(X)}$$

What about MAP / Regularization?

Prior $\mathrm{P}(\theta)$

Logistic Regression:

Observe X, Y → Learn $\mathrm{P}(\mathrm{Y}|\mathrm{X};\theta)$

- Parameterize:
  - $P(Y=1|X)=\sigma(\theta^T X)$, where $\sigma(z)=\frac{1}{1+e^{-z}}$ is the sigmoid function.
  - $P(Y=0|X)=1-P(Y=1|X)$
- Estimate $\hat{\theta}$ from observations:
  - $\hat{\theta}=\mathrm{argmax}_\theta \prod_i P(Y_i|X_i;\theta) P(\theta)$
    $=\mathrm{argmax}_\theta \sum_i [Y_i \log \sigma(\theta^T X_i)+(1-Y_i)\log(1-\sigma(\theta^T X_i))] - R(\theta)$
- Calculate $P(Y|X)$

Discriminative vs Generative Models

- Discriminative models optimize the conditional likelihood:

$$\widehat{\theta_{disc}} = \operatorname{argmax}_{\theta} P(Y|X;\theta)$$

- Generative models optimize the joint likelihood:

$$\widehat{\theta_{gen}} = \operatorname{argmax}_{\theta} P(X,Y;\theta)$$

Are these the same optimization?

Discriminative vs Generative Models

- Discriminative models optimize the conditional likelihood:

$$\widehat{\theta_{disc}} = \text{argmax}_\theta P(Y|X;\theta) = \text{argmax}_\theta \frac{P(X|Y;\theta)P(Y;\theta)}{P(X;\theta)}$$

- Generative models optimize the joint likelihood:

$$\widehat{\theta_{gen}} = \text{argmax}_\theta P(X,Y;\theta) = \text{argmax}_\theta P(X|Y;\theta)P(Y;\theta)$$

Are these the same optimization?

Same optimization when $P(X;\theta)$ is invariant to θ

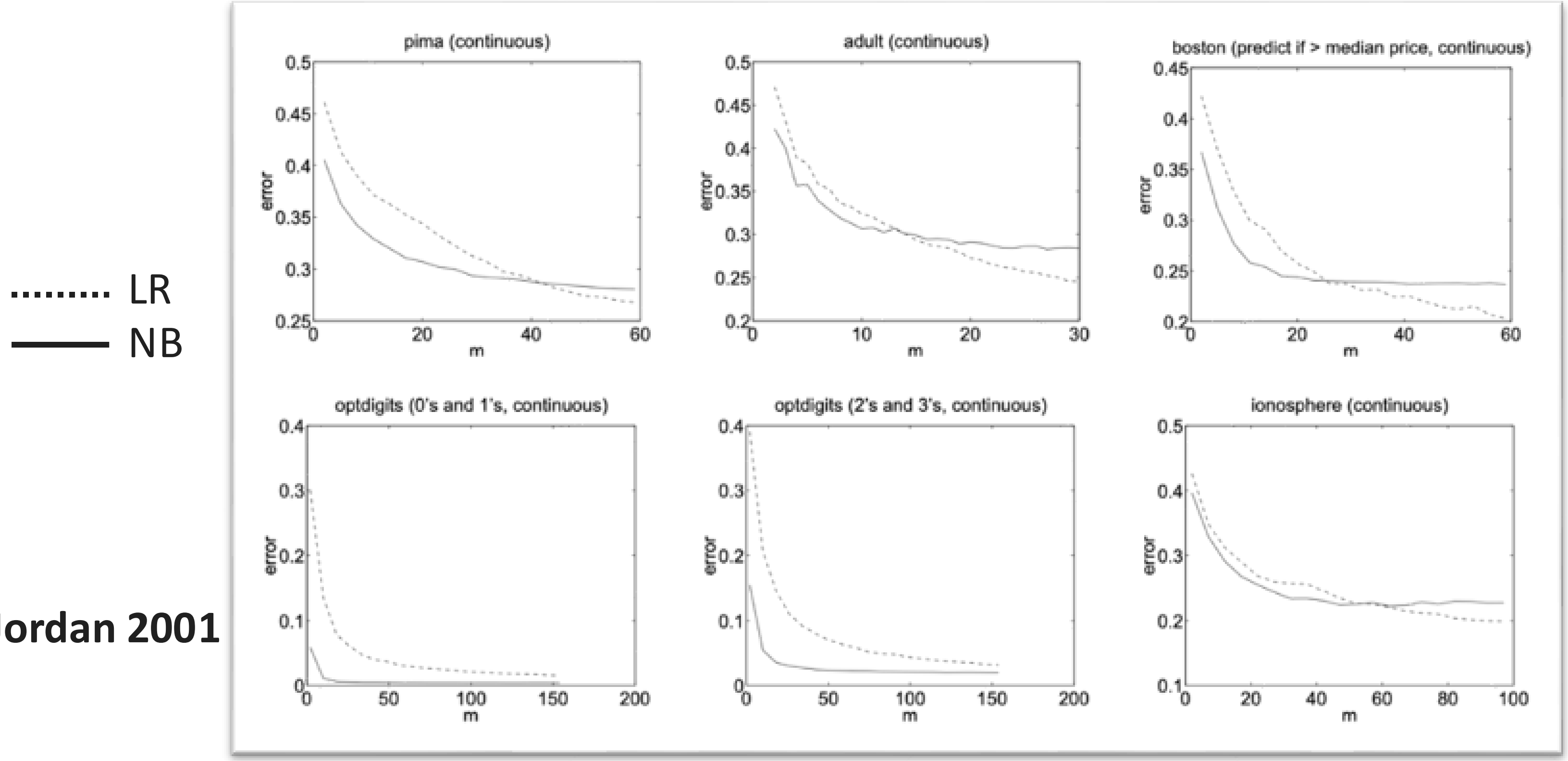
Logistic Regression vs Naïve Bayes

Logistic Regression	Naïve Bayes
Discriminative	Generative
Defines $P(Y\|X;\theta)$	Defines $P(X,Y;\theta)$
Estimates $\widehat{\theta_{lr}} = \text{argmax}_\theta P(Y\|X;\theta)$	Estimates $\widehat{\theta_{nb}} = \text{argmax}_\theta P(X,Y,\theta)$
Lower asymptotic error on classification	Higher asymptotic error on classification
Slower convergence in terms of samples	Faster convergence in terms of samples

Discriminative vs Generative: A Proposition

- “While discriminative learning has lower asymptotic error, a generative classifier may also approach its (higher) asymptotic error much faster.”

Why?



Ng & Jordan 2001

Discriminative vs Generative: A Proposition

- “While discriminative learning has lower asymptotic error, a generative classifier may also approach its (higher) asymptotic error much faster.”
- Underlying assumption of this statement:
  - Generative models of the form $\boldsymbol{P(X, Y, \theta)}$ make **more simplifying assumptions** than do discriminative models of the form $\boldsymbol{P(Y|X, \theta)}$.
  - **Not always true**
  - “So far there is no theoretically correct, general criterion for choosing between the discriminative and the generative approaches to classification of an observation $\mathbf{x}$ into a class y; the choice depends on the relative confidence we have in the correctness of the specification of either $p(y|\mathbf{x})$ or $p(\mathbf{x}, y)$ for the data.”

Xue & Tittering 2008

Modern Deep Generative Models (DGMs)

- Goal: Generative models of the form $\boldsymbol{P(X, Y, \theta)}$ without strong simplifying assumptions.
- Hidden structure z that explains high-dim. x
- Fundamental challenge: We never observe z
- This makes two core computations difficult:
  - **Marginal likelihood:** $p_\theta(x) = \int p_\theta(x, z) dz$
  - **Posterior inference:** $p_\theta(z \mid x) \propto p_\theta(x \mid z) p(z)$
- Each type of DGM makes a tradeoff

Coming up…

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9	10/27, 10/29	A Linear Intro to Generative Models, Factor Analysis, Autoencoders, VAEs	
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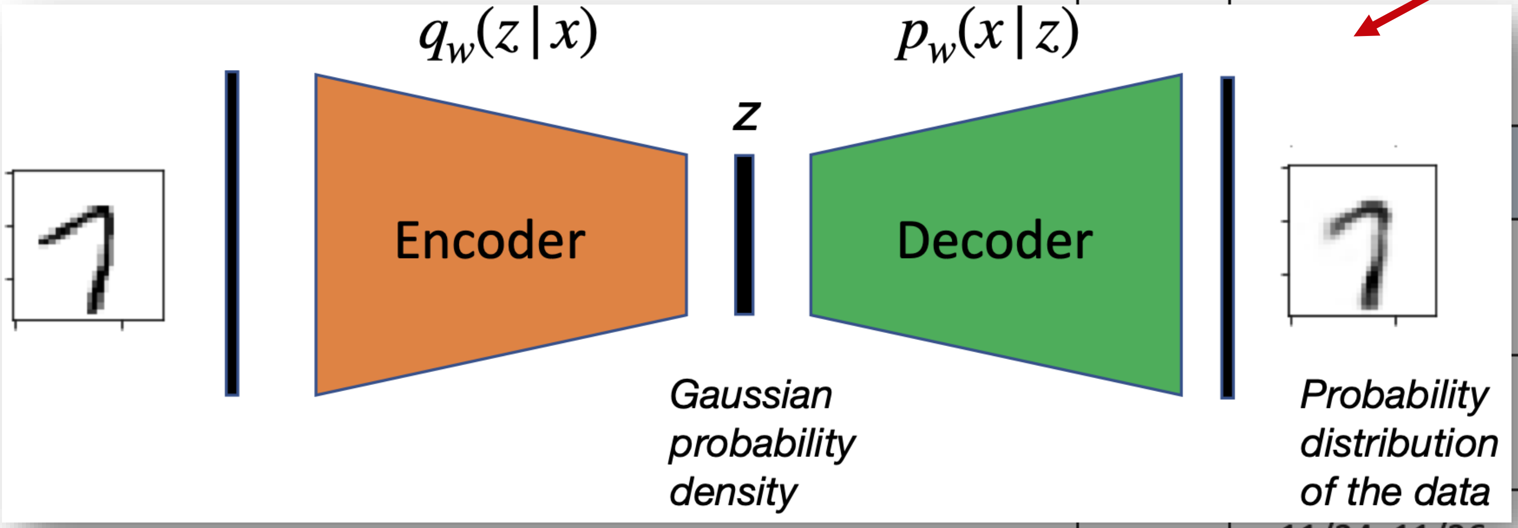

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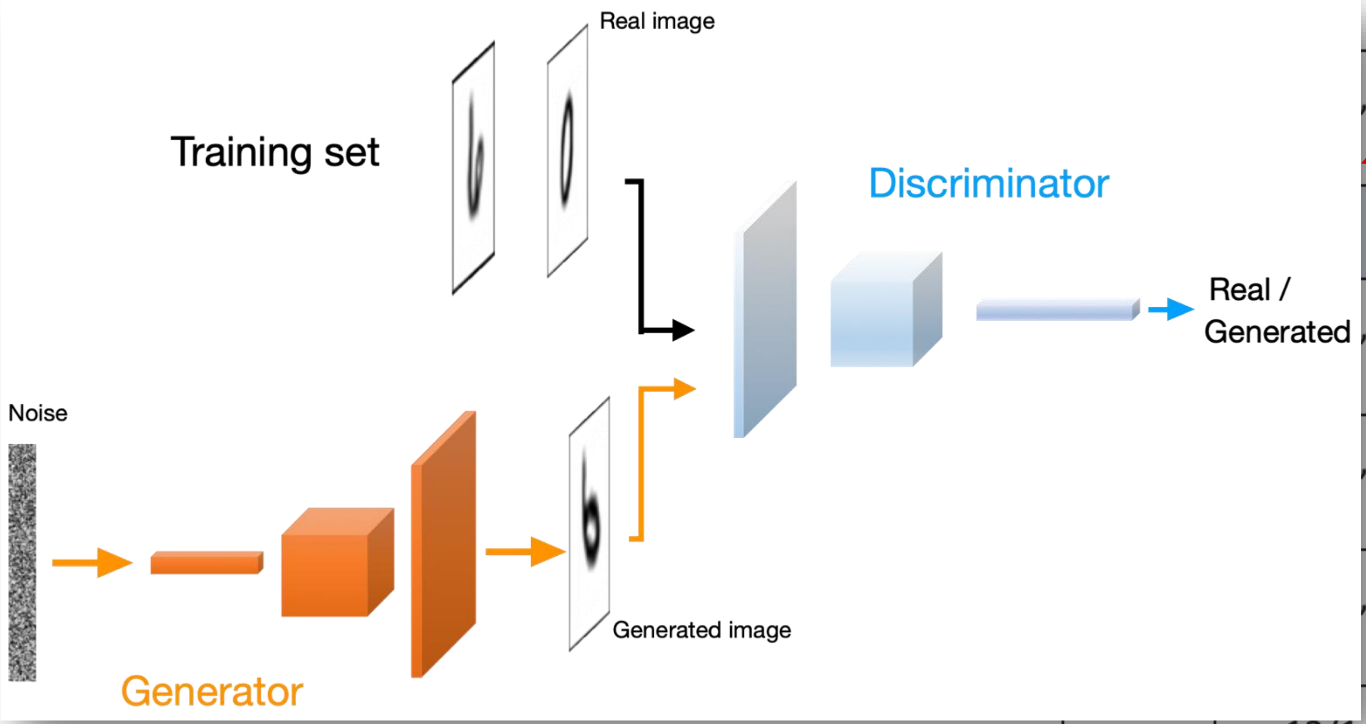

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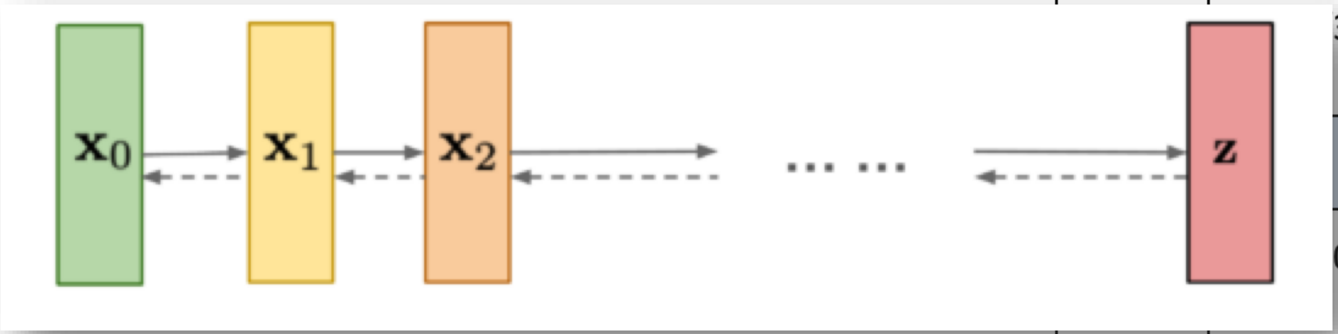

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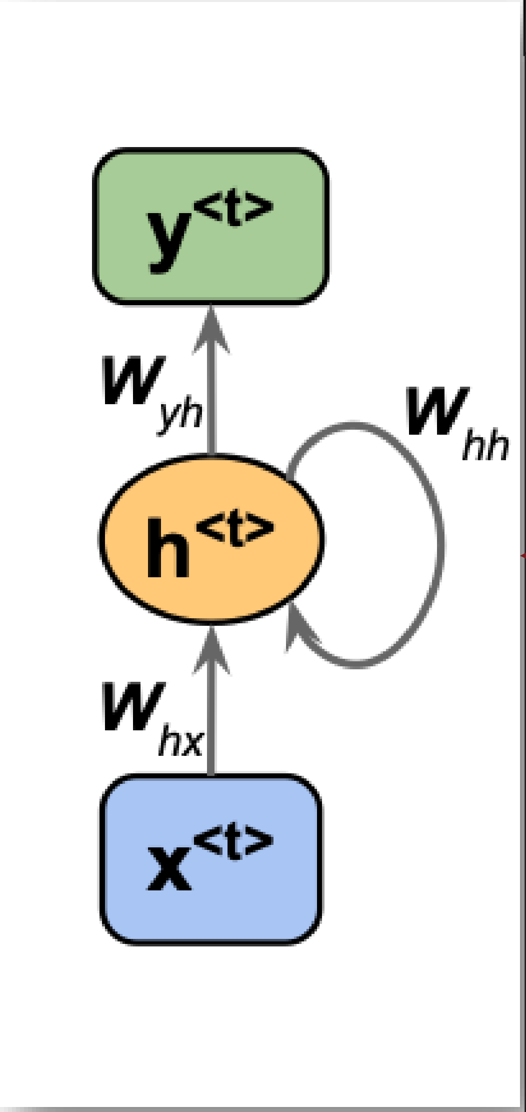

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Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L. and Polosukhin, I., 2017. Attention Is All You Need.

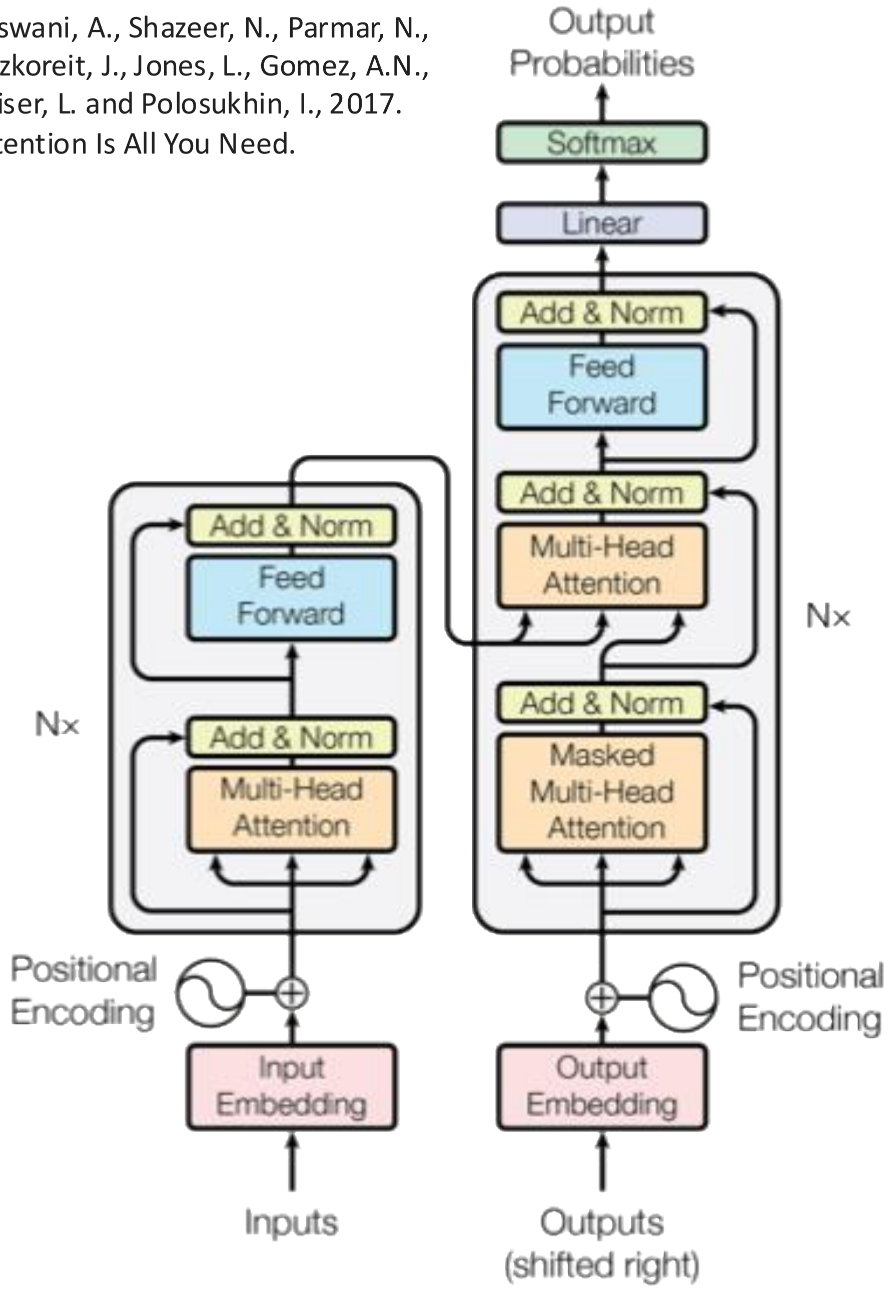

Figure 1: The Transformer - model architecture.

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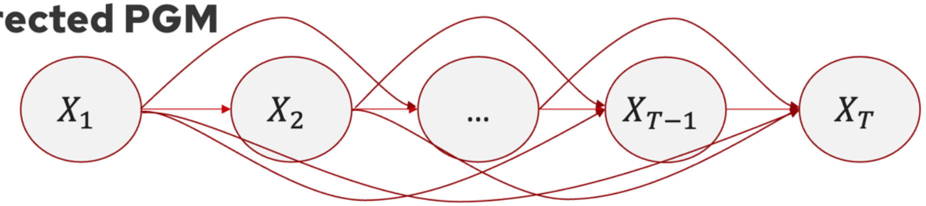
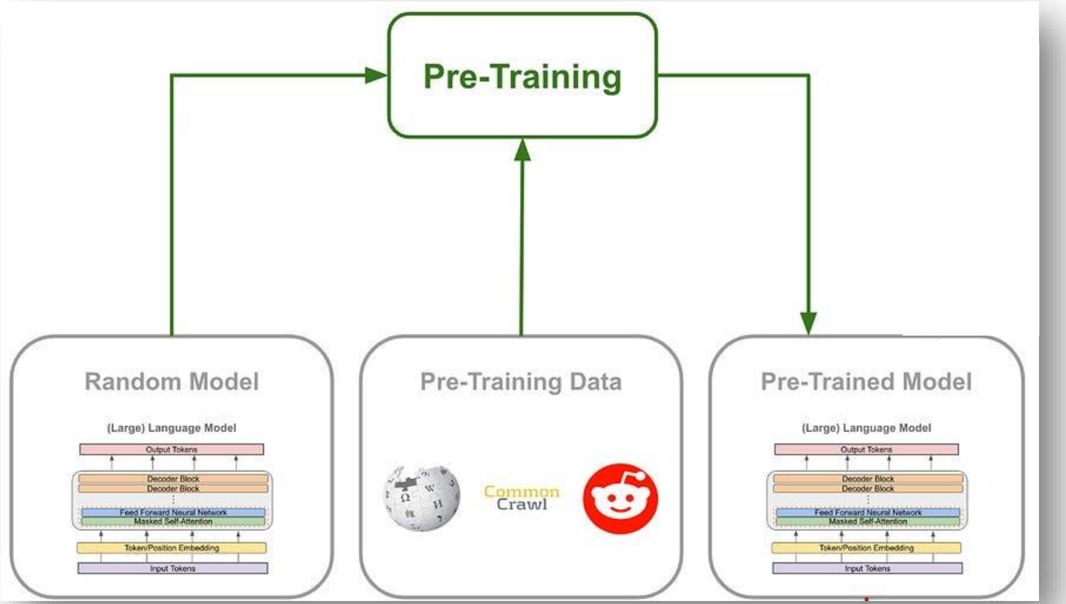
GPT = Probabilistic Model + Transformer Decoder

- Directed PGM



$$P_\theta(X) = \prod_i \prod_t P_\theta(X_{i,t} \mid X_{i,<t})$$

Coming up…



https://cameronrwolfe.substack.com/p/understanding-and-using-supervised

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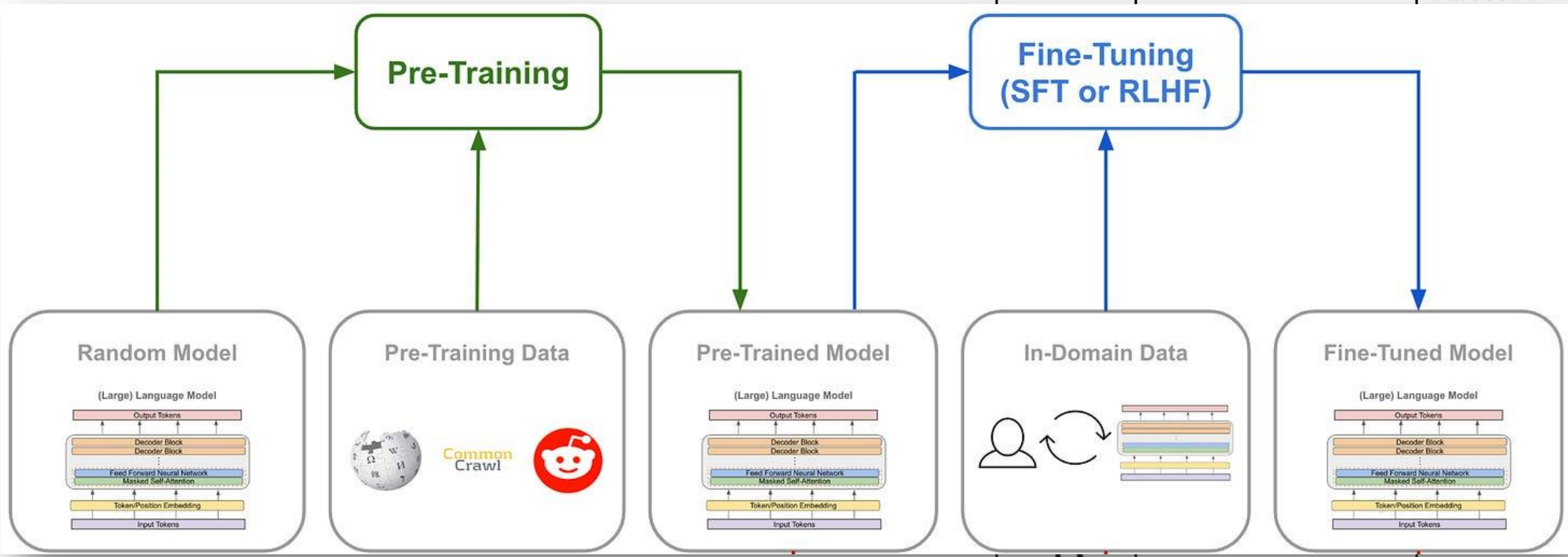

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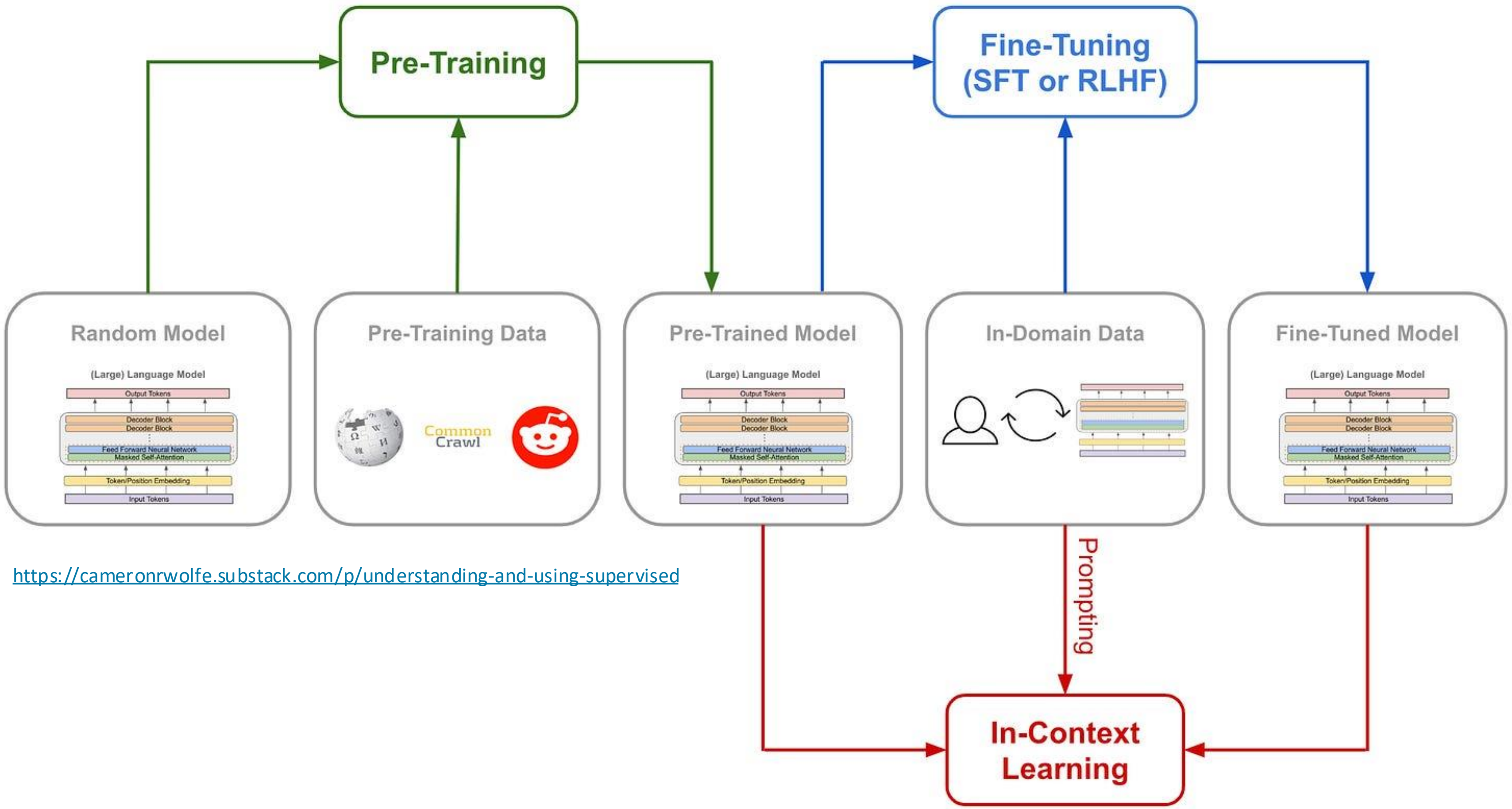

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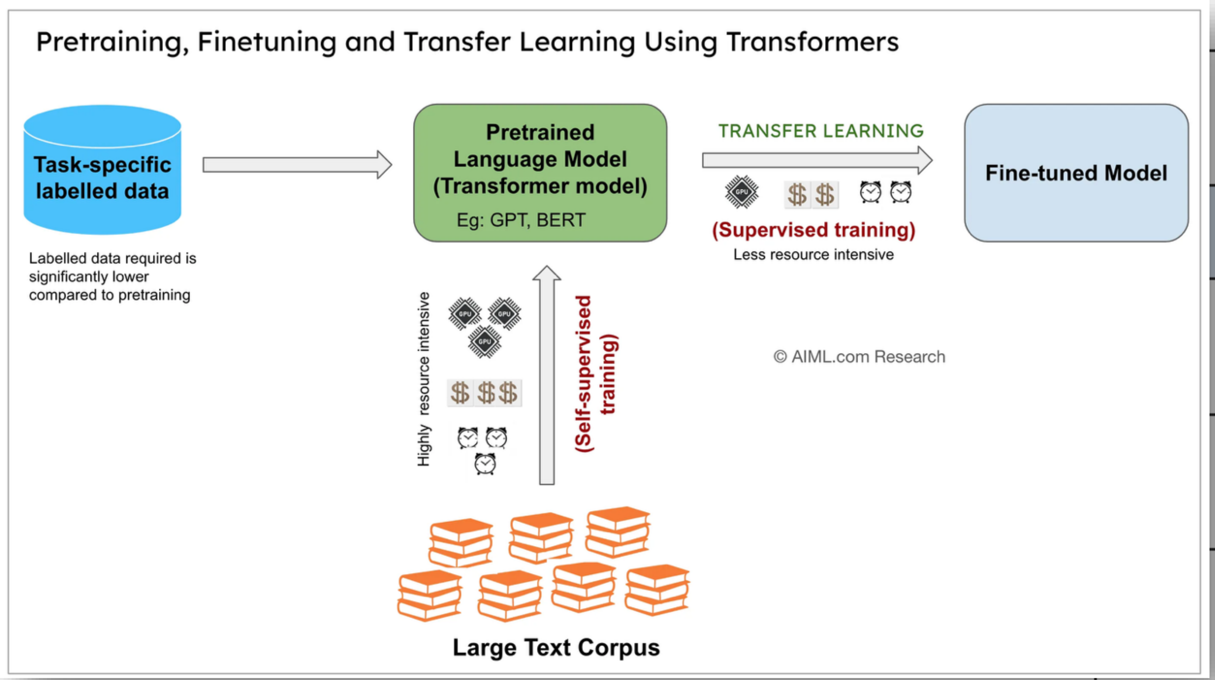

Source: AIML.com Research

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Questions?

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