STAT 453: Introduction to Deep Learning and Generative Models

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Lecture 18: Diffusion Models

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Reading: See course homepage

Project

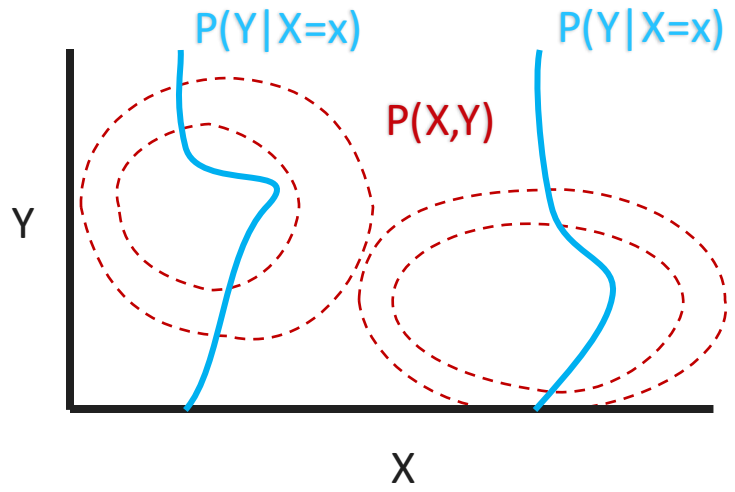
- https://adaptinfer.github.io/dgm-fall-2025/project/

Today

- Diffusion Models

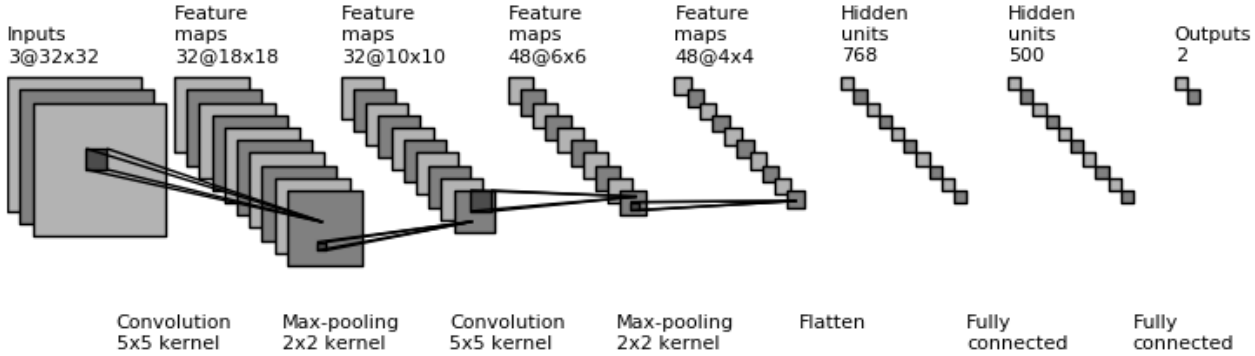
Generative and Discriminative Models

- **Generative**:
  - Models the joint distribution $\mathrm{P}(X, Y)$.
- **Discriminative**:
  - Models the conditional distribution $\mathrm{P}(Y|X)$.



Where we're going: Deep Generative Models

Discriminative Model (what we've seen so far)



Cat or dog?

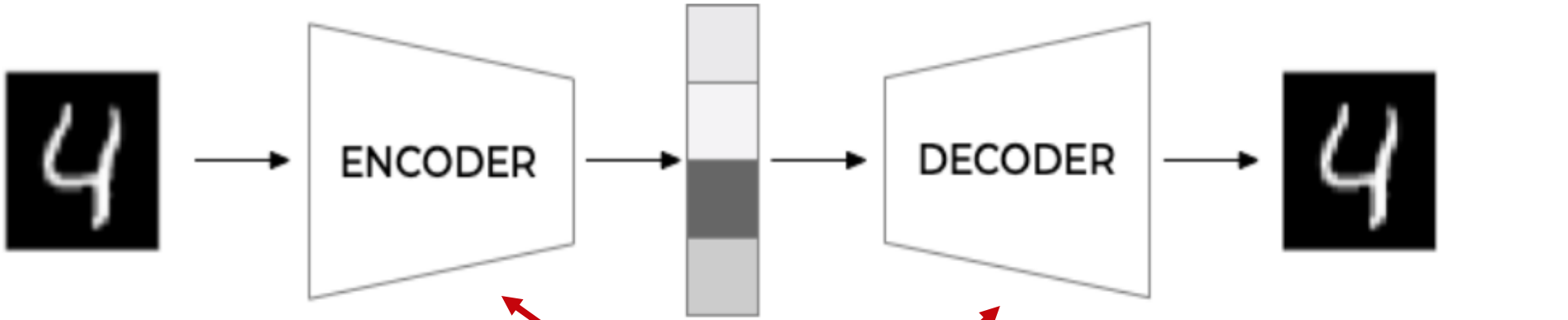
Generative Model (what we're going to see)

Gemini Grok deepseek

Modern Deep Generative Models (DGMs)

- Goal: Generative models of the form $\boldsymbol{P}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta})$ without strong simplifying assumptions.
- Hidden structure z that explains high-dim. x
- Fundamental challenge: We never observe z
- This makes two core computations difficult:
  - **Marginal likelihood:** $p_\theta(x) = \int p_\theta(x, z) dz$
  - **Posterior inference:** $p_\theta(z \mid x) \propto p_\theta(x \mid z) p(z)$
- Each type of DGM makes a tradeoff

Autoencoders



$$\tilde{x} = f(h) = f\big(g(x)\big)$$

[Michelucci 2022]

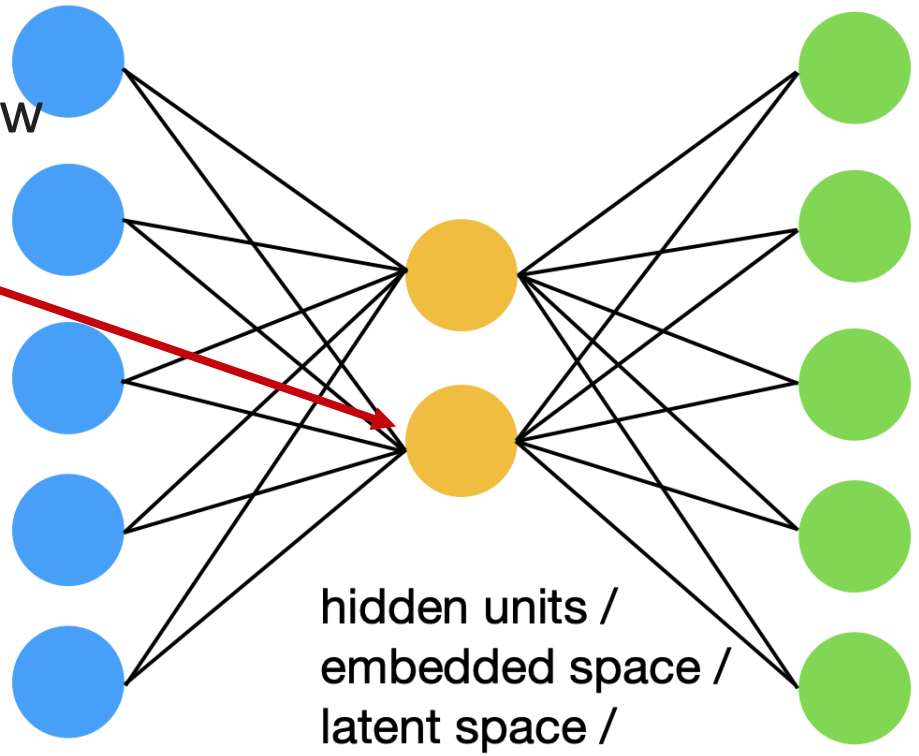
Variational Autoencoders

Kullback-Leibler divergence term
where $p(z) = \mathcal{N}\left(\mu = 0, \sigma^2 = 1\right)$

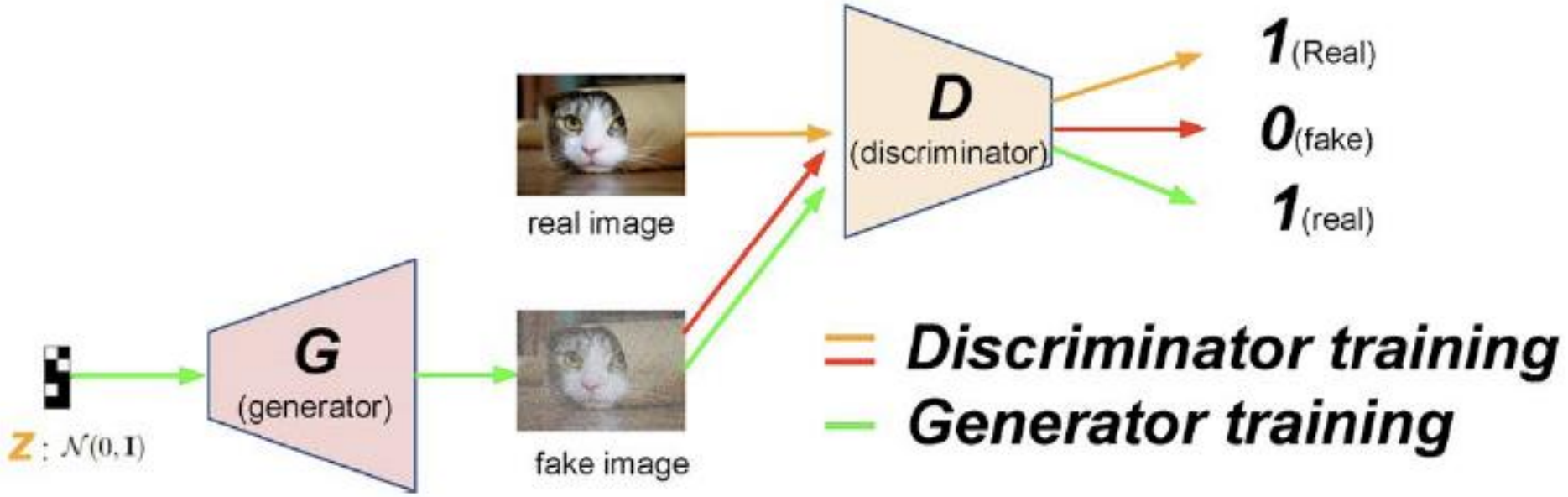
$$L^{[i]} = -\mathbb{E}_{z \sim q_w(z|x^{[i]})}\left[\log p_w\left(x^{[i]}|z\right)\right] + \mathbf{KL}\left(q_w\left(z|x^{[i]}\right)||p(z)\right)$$

Define latent dim to follow Normal distribution

Enables sampling



Generative Adversarial Networks



Discriminator: $\max_D \mathcal{L}_D = \mathbb{E}_{\boldsymbol{x} \sim p_{data}(\boldsymbol{x})} [\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x} \sim G(\boldsymbol{z}), \boldsymbol{z} \sim p(\boldsymbol{z})} [\log(1 - D(\boldsymbol{x}))]$

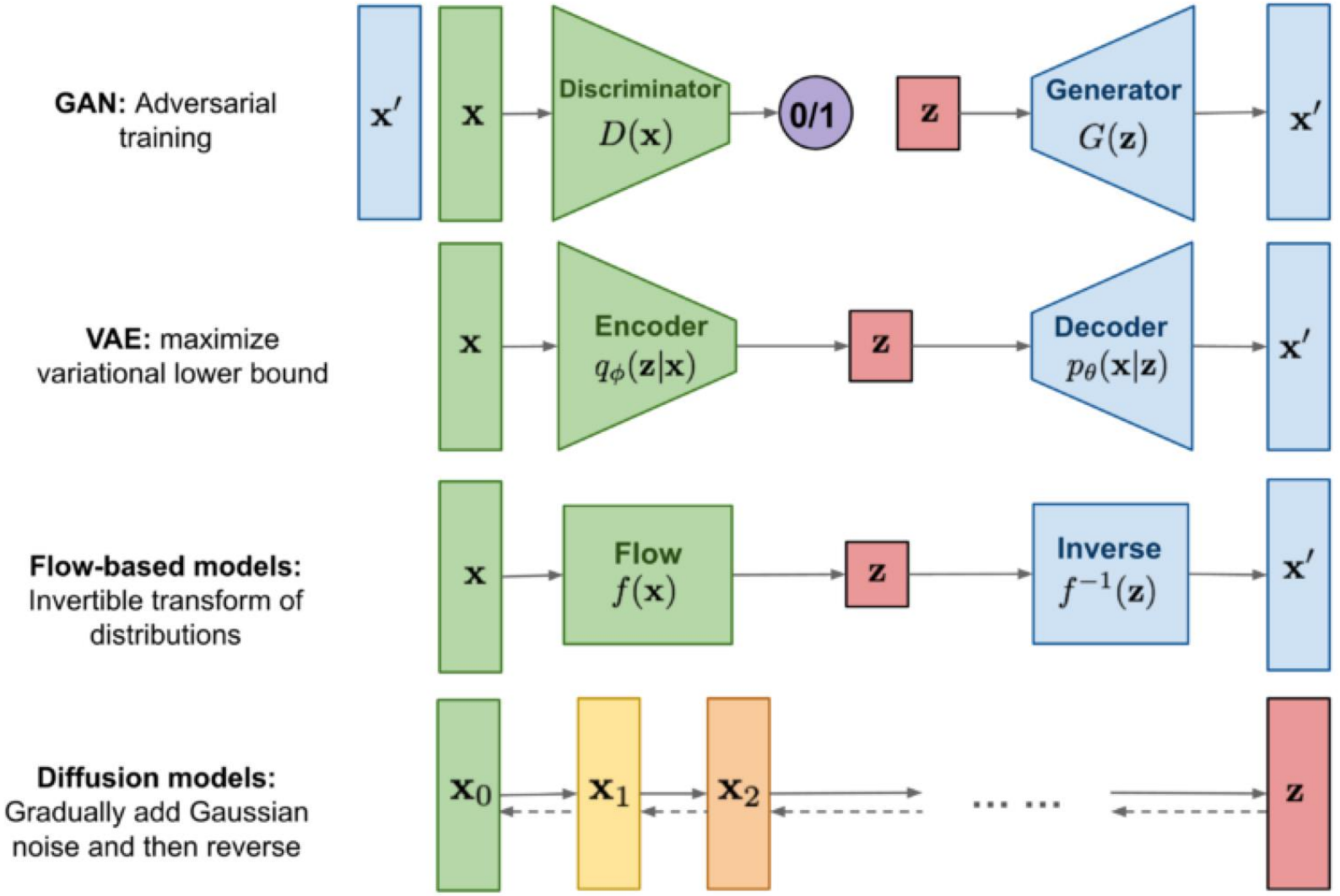
Generator: $\min_G \mathcal{L}_G = \mathbb{E}_{\boldsymbol{x} \sim G(\boldsymbol{z}), \boldsymbol{z} \sim p(\boldsymbol{z})} [\log(1 - D(\boldsymbol{x}))]\,.$

Summary

Property	VAE	GAN
What we specify	Prior p(z), Likelihood $p_\theta(x \mid z)$	Prior p(z), Generator $G_\theta(z)$
Induced $p(x)$	$p_\theta(\mathrm{x}) = \int_z p_\theta(X \mid z)p(z)dz$	$p_\theta(\mathrm{x}) = \int_z p_\epsilon\big(x - \mathrm{G}_\theta(\mathrm{z})\big)\mathrm{p}(\mathrm{z})\mathrm{dz}$
Simplifying assumption	Choose a **restricted** variational posterior $q_\phi(z \mid x)$	**Replace NLL** with a **distributional discrepancy** on samples (adversarial/IPM).
Training objective	ELBO: $E_q[\log p_\theta(x \mid z)] - KL(q_\phi(z \mid x) \big\| p(z))$	Minimax fooling discriminator
What's ignored from $p_\theta(x)$	$KL(q_\phi(z \mid x) \big\| p_\theta(z \mid x))$	**All of NLL**: $\log p_\theta(x)$isn't evaluated or maximized.
Modes	Covering	Collapse
Generated Samples	Blurry	Realistic
Training	Relatively robust	Fragile

Diffusion Models

Overview and comparison of generative models

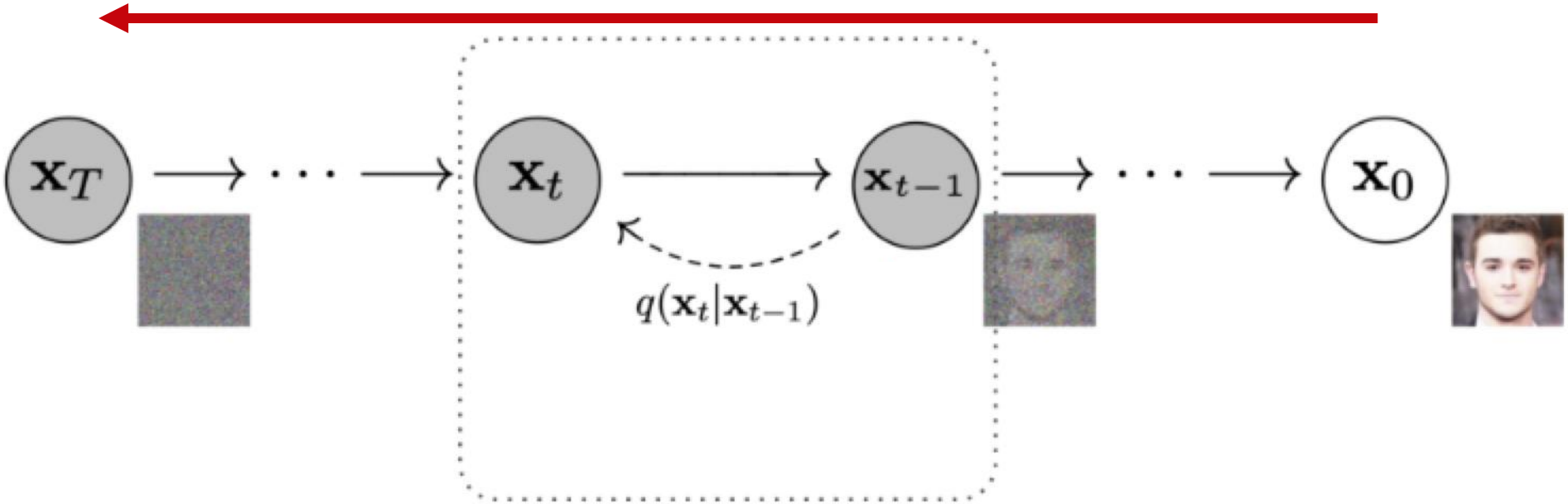


Source: https://lilianweng.github.io/posts/2021-07-11-diffusion-models/

Diffusion

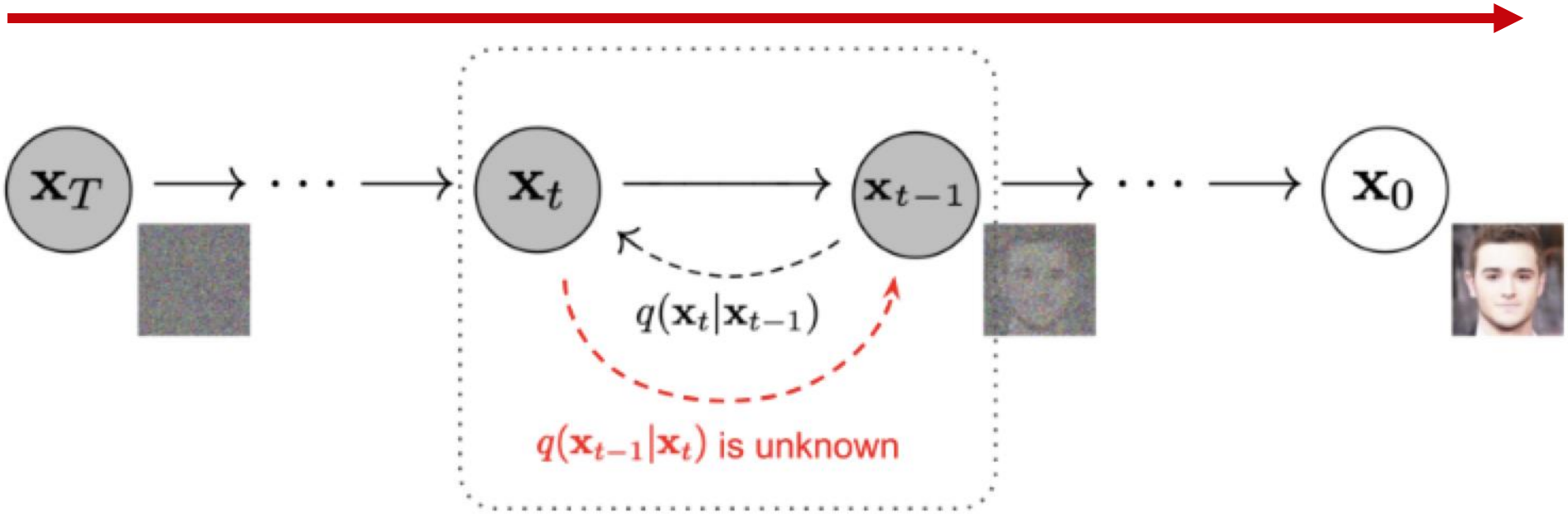
Diffusion models: forward pass

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{1-\beta_t}\mathbf{x}_{t-1}, \beta_t\mathbf{I}) \quad q(\mathbf{x}_{1:T}|\mathbf{x}_0) = \prod_{t=1}^{T} q(\mathbf{x}_t|\mathbf{x}_{t-1})$$



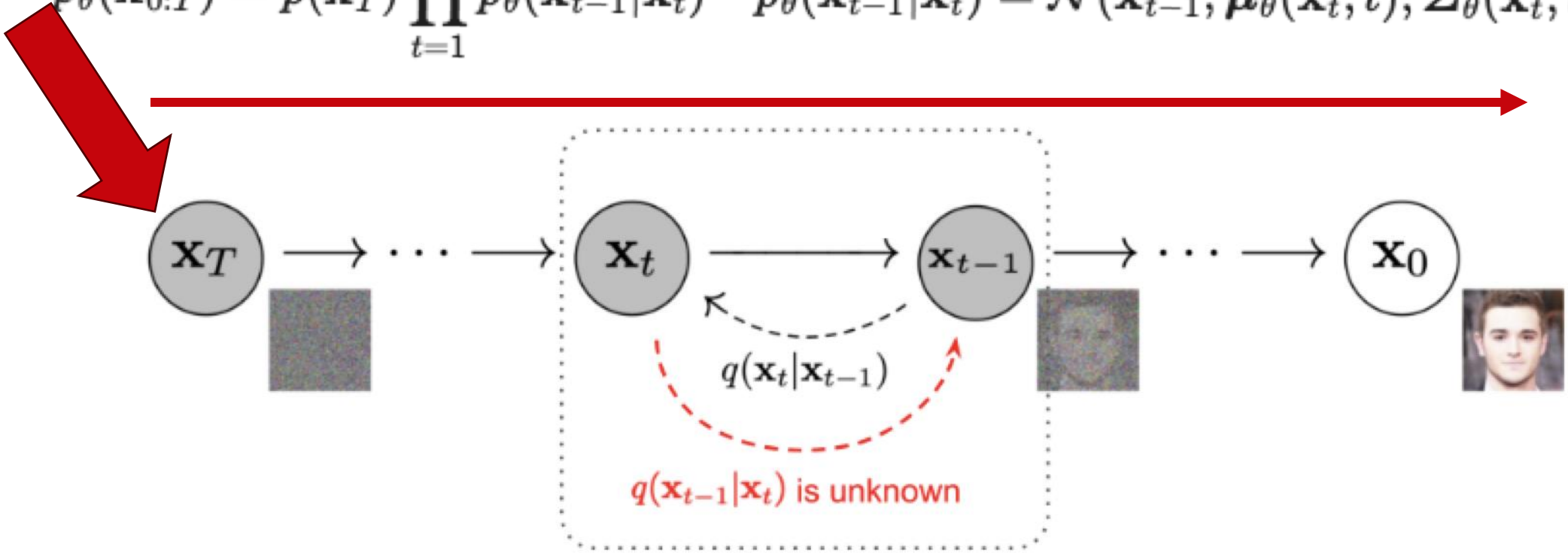
Diffusion models: reverse pass

$$p_\theta(\mathbf{x}_{0:T}) = p(\mathbf{x}_T) \prod_{t=1}^{T} p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) \quad p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t))$$

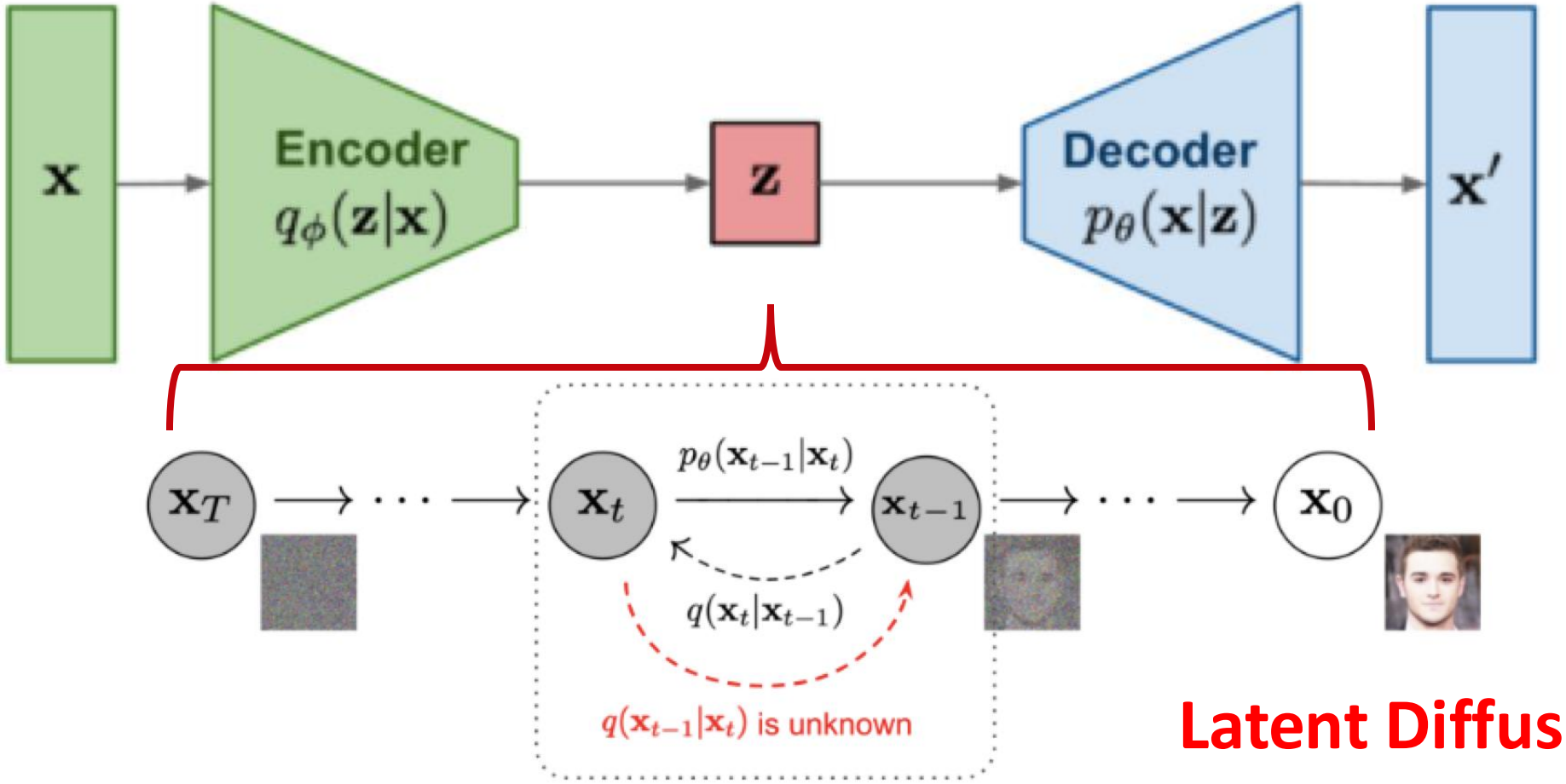


Diffusion models: generating a new sample

$$p_\theta(\mathbf{x}_{0:T}) = p(\mathbf{x}_T) \prod_{t=1}^{T} p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) \quad p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t))$$

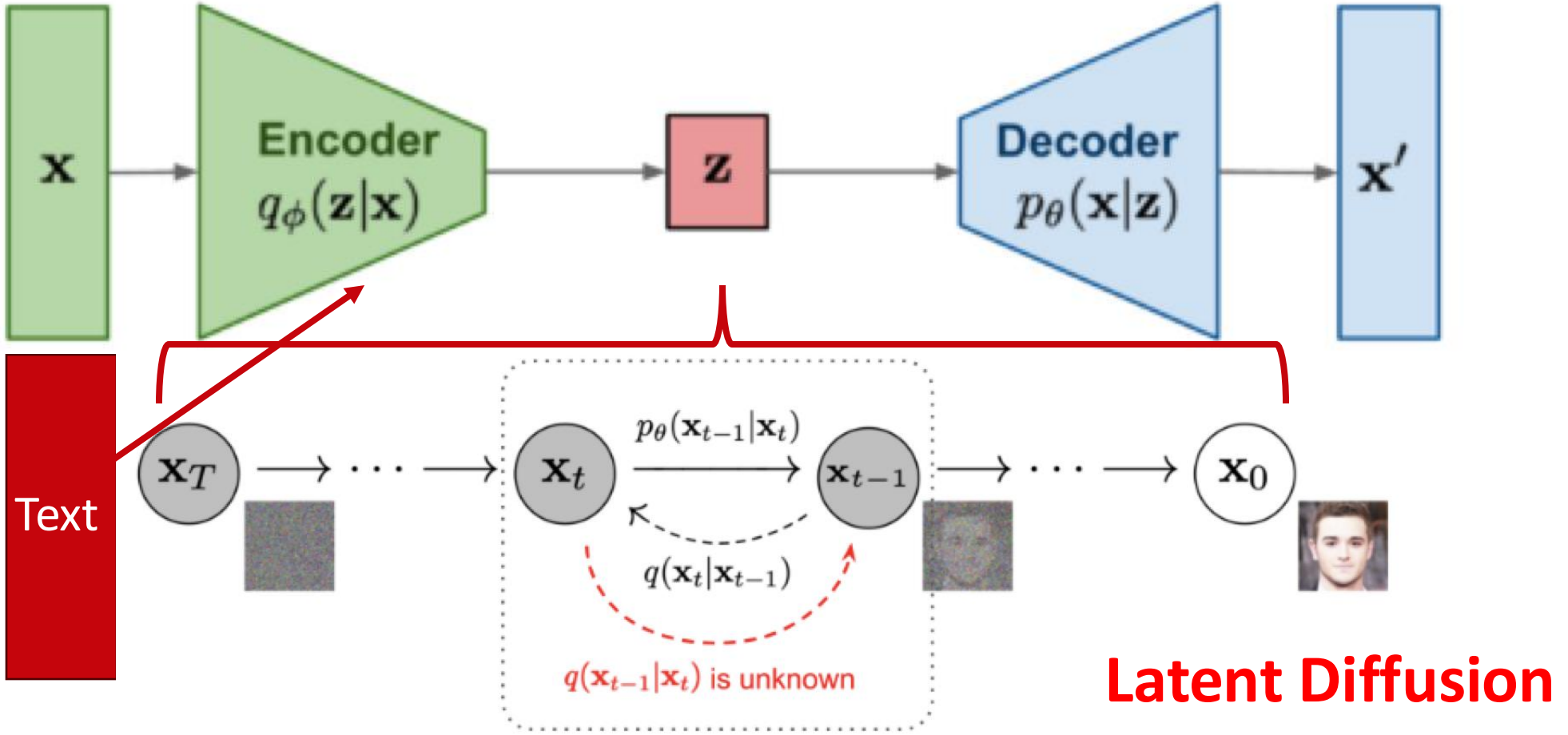


Should we really run this process on pixels?



Latent Diffusion

Stable Diffusion: Add Text Conditioning



Latent Diffusion

Stable Diffusion: Modern Image Generators

More reading

https://lilianweng.github.io/posts/2021-07-11-diffusion-models/

https://lilianweng.github.io/posts/2021-07-11-diffusion-models/

https://theaisummer.com/diffusion-models/

Property	VAE	GAN	Diffusion
What we specify	Prior p(z), Likelihood $p_\theta(x \mid z)$	Prior p(z), Generator $G_\theta(z)$	Fixed **forward noising** $q(x_t \mid x_{\{t-1\}})$; learn reverse $\mathrm{p}_\theta(x_{t-1} \mid x_t)$
Induced $p(x)$	$p_\theta(\mathrm{x}) = \int_z p_\theta(X \mid z)p(z)dz$	$p_\theta(\mathrm{x}) = \int_z p_\epsilon(x - \mathrm{G}_\theta(\mathrm{z}))\mathrm{p}(\mathrm{z})\mathrm{dz}$	$p_\theta(\mathrm{x}) = \int p(x_T)\prod_t p_\theta(x_{t-1} \mid x_t)dx$
Simplifying assumption	Choose a **restricted** variational posterior $q_\phi(z \mid x)$	**Replace NLL** with a **distributional discrepancy** on samples (adversarial/IPM).	Fix forward noise q; and optimize a **variational bound** on $-\log p_\theta(x_0)$.
Training objective	ELBO: $E_q[\log p_\theta(x \mid z)] - KL(q_\phi(z \mid x)\|p(z))$	Minimax fooling discriminator	**VLB / score matching**: with Gaussian schedules reduces to $\mathbb{E}_{t,x_0,\epsilon}[w(t) \parallel \epsilon - \epsilon_\theta(x_t, t) \parallel^2]$
What's ignored from $p_\theta(x)$	$KL(q_\phi(z \mid x)\|p_\theta(z \mid x))$	**All of NLL**: $\log p_\theta(x)$isn't evaluated or maximized.	Exact NLL not computed; optimize a **variational upper bound on NLL** (equivalently lower bound on $\log p$;(practical losses often **reweight** or drop constants from the exact VLB.
Modes	Covering	Collapse	Covering

Questions?